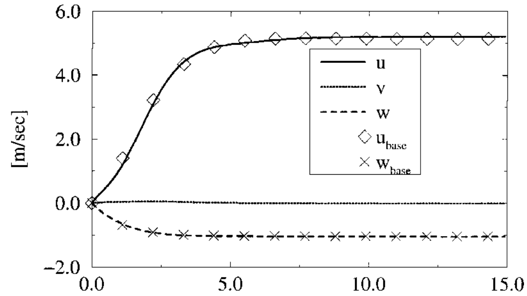
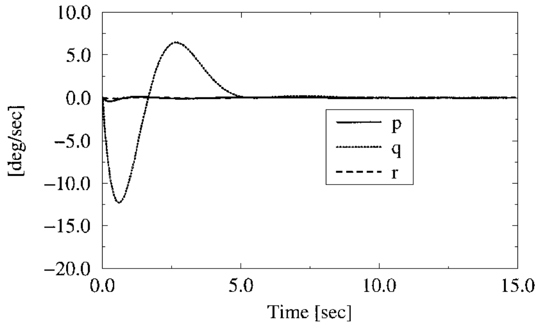
The gain matrices $\boldsymbol{H}$ and $\boldsymbol{K}$ are computed as

$$\boldsymbol{H} = \begin{bmatrix} 11.4274 & -0.9991 & -0.0076 & 2.9247 \\ 0.4293 & 5.0924 & 0.8072 & 1.0563 \\ -0.0593 & -0.2245 & -5.3610 & 0.1033 \\ 0.7600 & 2.5943 & -10.1460 & -11.4807 \end{bmatrix} \tag{10}$$

$$\boldsymbol{K} = \begin{bmatrix} 10.8358 & -0.8842 & -0.5380 & -1.1193 & -4.0450 & 2.5219 & -0.5852 & -9.1572 \\ 0.4481 & 3.7913 & 0.3816 & 1.3325 & -0.7996 & 1.1609 & 3.8849 & -0.3396 \\ -0.2041 & -0.1537 & -3.5639 & -0.0987 & -0.0227 & -0.0796 & -0.2615 & 0.2123 \\ -0.4035 & 0.1604 & -6.8850 & 1.1642 & -0.2847 & -10.4001 & 1.7214 & -0.2575 \end{bmatrix} \tag{11}$$




**a) Translational rates**




**b) Angular rates**

**Fig. 2 Response of the closed-loop system to a combinational speed command.**

The resultant closed-loop system is tested for a slow takeoff procedure. The forward and vertical velocities are commanded to 10 kn (5.14 m/s) and 2 kn (1.03 m/s), respectively. It should be noted that this test case is rather generic and not directly related to the base responses that were used in the design process. Figure 2a shows the resultant translational velocities. The primary axis values matched well with the base model response ($u_{\text{base}}$, $w_{\text{base}}$) while the off-primary axis response is decoupled. Figure 2b shows the angular rate responses. Deviation in the pitch rate is significantly large compared to the roll and yaw responses, which is natural for this type of maneuvering. Figure 2b also shows that the system is well decoupled, which results in the negligible roll and yaw rates.

## Conclusion

A new design method that utilizes the desired time history and an identification algorithm is proposed. The technique was applied to designing a helicopter control system for translational velocity command with zero yaw rate. The feedback and feedforward gains were found to make the system behave like a reference model. The resultant closed-loop system was tested for a slow takeoff case. The system response of the primary channel was very close to that of the reference model, and the off-axis response was negligible.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# Solving Control Allocation Problems Using Semidefinite Programming


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## I. Introduction

THE capabilities of modern combat and civilian aircraft keep increasing. In particular, modern-day aircraft have many available control surfaces and thrust vectoring capabilities that offer significant advantages over conventional architectures based on three control surfaces only, including reduced electromagnetic signature, tailless designs, energy-efficient maneuvering, and most importantly, much needed redundancy in case of battle damage. The trend toward the presence of multiple actuators in modern aircraft

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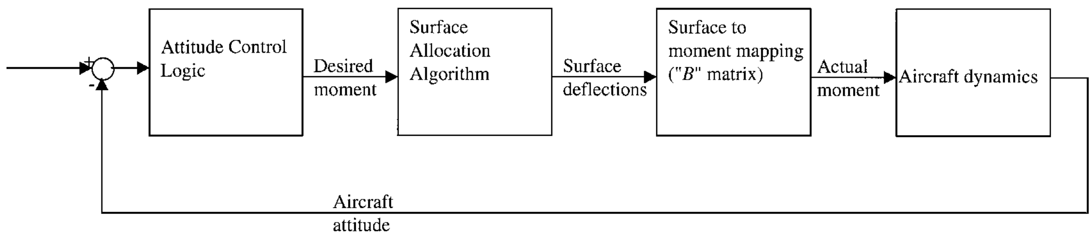


**Fig. 1 Modular architecture for aircraft autopilot.**

is likely to continue, with the advent of distributed actuation systems based, for example, on micro electro mechanical systems. The control allocation problem is to find a harmonious way to manage several actuators together to produce desired effects (usually moments) on the aircraft. The requirement for simplicity enables the reduction of software development costs by reusing existing control architectures as much as possible.

Several approaches to the control allocation problem exist. One of the main drivers for control surface allocation is to account for and avoid control surface deflection saturation and/or rate saturation. By many accounts, this problem may be handled directly by some of the recent advances made in the control of systems subject to saturations of many kinds. Assuming the plant is linear, Refs. 1–9 propose a number of approaches where actuator saturation is explicitly accounted for while attempting not to degrade system performance. Although theoretically attractive, these approaches are not implemented in practice yet and would require an overhaul of the control architecture currently in use on board modern fighter aircraft, possibly resulting in high software development costs.

A more modular architecture, compatible with current aircraft control systems is given in Fig. 1. In this architecture, the aircraft control algorithm specifies the desired set of moments to be applied to the aircraft, and a separate control surface allocation algorithm maps the desired moment to specific surface deflections. This architecture easily extends to applied forces if these are deemed important. The set of moments applied to the aircraft by the control surfaces is usually small enough that it may be considered to be linear in the control surface deflections at a given flight condition (parameterized as a function of angle of attack, Mach number, and dynamic pressure). Denoting $u$ as the control surface deflections and $m$ the actual moment, we write

$$m = Bu \tag{1}$$

where $m \in \boldsymbol{R}^3$ and $u \in \boldsymbol{R}^p$, where $p$ is much larger than 3. In typical modern-day civilian and combat aircraft, $p$ can reach 20 or more. It is assumed $B$ is full rank. In addition, the control variable $u$ is assumed to be bounded above and below:

$$u_{i,\min} \leq u_i \leq u_{i,\max}, \qquad i = 1, \ldots, p \tag{2}$$

Using linear scalings and shifting the origin, it is always possible to recast the problem to the case when $u_{i,\min} = -1$ and $u_{i,\max} = 1$ $(i = 1, \ldots, p)$. We will therefore restrict our attention to this case only.

The moment allocation problem is then as follows: Given a desired moment $m_d$, find a set of surface deflections $u$ such that $Bu = m_d$. It is straightforward to see that the set of moments that may be achieved with such controls is a three-dimensional polytope and that the moment allocation problem can be handled by solving a linear program[10,11]: Denoting $m_d = [m_{1,d}, m_{2,d}, m_{3,d}]^T$, the desired moment (assumed to be achievable by some combination of control surface deflections), then the following linear program:

$$\begin{aligned} &\text{Minimize} \quad \sum_{j=1}^{3} |m_j - m_{j,d}| \\ &\text{Subject to} \quad (1), (2) \end{aligned} \tag{3}$$

yields a desired set of controls. However, linear programming suffers from high computational costs as well as high software development costs (i.e., from the viewpoint of real-time control logic). Although this situation is currently being examined and improved upon,[12] it is likely to represent an ongoing significant challenge to aircraft's onboard computers, especially older-type aircraft with many years of service to come but slower computer architectures. As a consequence, many other approaches have been proposed to solve the allocation problem approximately but faster. For example, Bordignon and Durham[13] have considered linear maps for surface allocation:

$$u = Qm_d \tag{4}$$

where $Q$ is a pseudoinverse to $B$ (i.e., $BQ = I$). The latter condition ensures that the produced moment is the desired one indeed, at least as far as numerical models are concerned. This form of control allocation technique is particularly attractive from a computational standpoint because it is a linear mapping and computationally tractable for existing onboard computers. It was shown[13] that the main issue is to choose $Q$ so as to maximize the size of the set of desired moments $m_d$ for which the mapping (4) does not saturate the control $u$. In particular, one would like to find a linear mapping such that this set is as close as possible to the set of moments that may be attained using any combination of the control surfaces deviations (e.g., via linear programming). It was shown by the authors that the Moore-Penrose inverse of $B$ is often a very poor choice for $Q$ and that finding the optimal mapping is sometimes challenging and computational approaches may find local optima only.

Motivated by this approach, we present an alternate approach to compute good linear mappings $Q$ for control allocation purposes. The approach is based on semidefinite programming, recently introduced to the control community as a powerful means to solve many control problems.[14] We demonstrate the approach on the control allocation problem for a numerical model of the F-18 High Alpha Research Vehicle (HARV) and compare previously known approaches to that problem.

## II. Control Allocation Procedure

The proposed approach to control allocation via linear mappings is based on finding a linear mapping $Q$ from the moment space to the control space, so that $BQ = I$ and the set of moments yielding nonsaturating controls is maximized by volume. The following sets are of interest: Define first 8, the set of moments that may be generated by all possible actuator surface deflections. In other terms,

$$\begin{aligned} 8 = \{m \in \boldsymbol{R}^3 \text{ so that there exists} \\ u \in \boldsymbol{R}^m \text{ and } \|u\|_\infty \leq 1 \text{ and } m = Bu\} \end{aligned}$$

where $\|u\|_\infty$ is the maximum absolute value of the coefficients of the vector $\boldsymbol{u}$. Define then $8_Q$, the set of moments that map onto nonsaturating controls via the moment allocation mapping $Q$, i.e.,

$$8_Q = \{m \in \boldsymbol{R}^3 \text{ so that } \|Qm\|_\infty \leq 1\}$$

Both 8 and $8_Q$ are polytopes, and it is easy to show that $8_Q \subseteq \forall$. Moreover both 8 and $8_Q$ are symmetric with respect to the origin.[14] The problem under consideration in this Note is to maximize the volume of $8_Q$ over all possible control surface allocation mappings $Q$ satisfying $BQ = I$. This task is generally considered to be highly nonlinear and difficult to solve.

Consider the following suboptimal approach instead: For any $Q$, consider the unique maximum volume ellipsoid contained in $8_Q$.[15] It is well known that this ellipsoid provides a good and simple approximation of the polytope $8_Q$ itself.[14] The proposed approach is thus to compute the maximum volume ellipsoid contained in $8_Q$

over all possible ellipsoids and all possible surface allocation mappings $Q$. We show this problem may be formulated as a convex optimization problem.

Consider first the problem of maximizing the volume of an ellipsoid contained in the polytope $8_Q$. Since $8_Q$ is symmetric, this ellipsoid is also centered around the origin[14,15] and the search may be done over ellipsoids of the form $\mathcal{E}_P$ centered around the origin and defined as

$$\mathcal{E}_P = \{m \in \boldsymbol{R}^3 \;\text{ such that } m^T P^{-1} m \leq 1\}$$

The volume of the ellipsoid $\mathcal{E}_P$ is proportional to log det $P$, a concave function of the entries of the matrix $P$.[16] The constraint that this

$$B^T = \begin{bmatrix} -4{:}832 & -53{:}30 & 1{:}100 \\ 4{:}832 & -53{:}30 & -1{:}100 \\ -5{:}841 & -6{:}486 & 0{:}3911 \\ 5{:}841 & -6{:}486 & -0{:}3911 \\ 1{:}674 & 0{:}000 & -7{:}428 \\ -6{:}280 & 6{:}234 & 0{:}000 \\ 6{:}280 & 6{:}234 & 0{:}000 \\ 2{:}920 & 0{:}001 & 0{:}030 \\ 0{:}001 & 35{:}53 & 0{:}001 \\ 1{:}000 & 0{:}001 & 14{:}85 \end{bmatrix} \times 10^{-2} \quad \text{corresponding to} \quad \begin{cases} \text{right horizontal tail} \\ \text{left horizontal tail} \\ \text{right aileron} \\ \text{left aileron} \\ \text{combined rudders} \\ \text{right trailing edge flap} \\ \text{left trailing edge flap} \\ \text{roll thrust vector vane} \\ \text{pitch thrust vector vane} \\ \text{yaw thrust vector vane} \end{cases} \tag{5}$$

ellipsoid must satisfy is to be contained in the polytope $8_Q$, which may be expressed from the definition of $8_Q$ as

$$\max_{m \in \mathcal{E}_P} \|Qm\|_\infty \leq 1$$

Denoting the rows of $Q$ as $q_1; q_2; \ldots; q_p$, and computing

$$\max_{m \in \mathcal{E}_P} |q_i m|^2 = q_i P q_i^T; \qquad i = 1; \ldots; p$$

the optimization problem becomes

$$\begin{aligned} &\text{maximize} && \log\det P \\ &\text{subject to} && q_i P q_i^T \leq 1; \qquad i = 1; \ldots; p \end{aligned}$$

This is a convex optimization problem in the coefficients of $P$, involving linear matrix inequality constraints, which may be solved easily using existing software.[17] Jointly optimizing over $P$ and $Q$ directly is not a convex optimization problem. However, introducing the new variable $R = QP$, and denoting the rows of $R$ as $r_1; r_2; \ldots; r_p$, the joint optimization problem may be written as an optimization problem over the variables $P$ and $R$

$$\begin{aligned} &\text{maximize} && \log\det P \\ &\text{subject to} && r_i P^{-1} r_i^T \leq 1; \qquad i = 1; \ldots; p \\ &&& BRP^{-1} = I \end{aligned}$$

where the last constraint is equivalent to the constraint $BQ = I$. Transforming the constraints via Schur complement techniques,[14] the problem may be equivalently written as

$$\begin{aligned} &\text{maximize} && \log\det P \\ &\text{subject to} && \begin{bmatrix} P & r_i^T \\ r_i & 1 \end{bmatrix} \geq 0; \qquad i = 1; \ldots; p \\ &&& BR = P \end{aligned}$$

This is now a convex problem in the coefficients of $P$ and $R$ involving linear matrix inequality constraints. The additional constraint $BR = P$ is a linear equality constraint and may be handled by eliminating variables. Denoting the optimal solutions to this problem as $R^*$ and $P^*$, the optimal mapping $Q^*$ is recovered by computing $Q^* = R^* P^{*-1}$.

Although the proposed approach concentrates mostly on the mapping $Q$ as its end product, the ellipsoid $\mathcal{E}_P$ (in conjunction with $Q$) may be used to refine some of the approaches proposed in the recent literature.[11]

## III. Numerical Example

In this section we illustrate the efficiency of the proposed approach on a numerical model of the F-18 HARV.[13] We will consider the following input matrix $B$ for 10 independent controls at 10,000 ft, Mach 0.3, and 12.5 deg angle of attack:

The control surface limits are given in Table 1.

Using the solution procedure proposed in the previous section, we get the following optimal solution:

$$P^* = \begin{bmatrix} 1{:}9878 & 0{:}0004 & -0{:}1239 \\ 0{:}0004 & 27{:}4454 & 0{:}0003 \\ -0{:}1239 & 0{:}0003 & 1{:}3606 \end{bmatrix} \times 10^{-2} \tag{6}$$

$$Q^* = \begin{bmatrix} -0{:}6699 & -0{:}5436 & 0{:}1654 \\ 0{:}6702 & -0{:}5436 & -0{:}1651 \\ -3{:}5566 & -0{:}2861 & 0{:}0242 \\ 3{:}5567 & -0{:}2862 & -0{:}0240 \\ 0{:}4124 & 0{:}0000 & -4{:}4236 \\ -3{:}1788 & 0{:}2273 & -0{:}2896 \\ 3{:}1788 & 0{:}2272 & 0{:}2895 \\ 3{:}7240 & 0{:}0000 & 0{:}3947 \\ 0{:}0002 & 0{:}9995 & 0{:}0003 \\ 0{:}4854 & -0{:}0000 & 4{:}4947 \end{bmatrix} \tag{7}$$

This solution results in an ellipsoid $\mathcal{E}_P^*$ with a volume of $3{:}600e-2$. Through the solution of a simple branch-and-bound optimization

**Table 1 Control surface limits**

| Control surface | Lower bound, rad | Upper bound, rad |
|---|---|---|
| Right horizontal tail | −0.4189 | 0.1833 |
| Left horizontal tail | −0.4189 | 0.1833 |
| Right aileron | −0.5236 | 0.5236 |
| Left aileron | −0.5236 | 0.5236 |
| Combined rudders | −0.5236 | 0.5236 |
| Right trailing edge flap | −0.1396 | 0.7854 |
| Left trailing edge flap | −0.1396 | 0.7854 |
| Roll thrust vector vane | −0.5236 | 0.5236 |
| Pitch thrust vector vane | −0.5236 | 0.5236 |
| Yaw thrust vector vane | −0.5236 | 0.5236 |

problem, we obtained the following upper and lower bounds on the volume of 8 :

$$9{:}070e{-}2 \le \mathrm{Vol}.\,8\,/ \le 9{:}152e{-}2 \quad (8)$$

and the following bounds on the volume of $8_{\,Q}$:

$$5{:}254e{-}2 \le \mathrm{Vol}.\,8_{\,Q}/ \le 5{:}466e{-}2 \quad (9)$$

Therefore the ratio of the volume of $\mathcal{E}$ to $8_{\,Q}$ was found to be bounded as

$$68{:}52\% \ge \frac{\mathrm{Vol}.\,\mathcal{E}_{P}/}{\mathrm{Vol}.\,8_{\,Q}/} \ge 65{:}86\% \quad (10)$$

and the ratio of the volumes of $8_{\,Q}$ to 8 was found to be bounded as

$$60{:}3\% \ge \frac{\mathrm{Vol}.\,8_{\,Q}/}{\mathrm{Vol}.\,8\,/} \ge 57{:}40\% \quad (11)$$

This ratio may be compared with a comparable volume ratio of 67:1% using the methods proposed by Bordignon and Durham[13] (an 11% difference in volume). Since the volumes represent distances cubed, the 57:43% and 67:1% volume ratios approximately represent distance ratios of 83:12% and 87:55%. This is a difference of 5:3%. Although the presented method does not achieve the same level of performance as earlier methods[13] (at least in the given numerical example), it still has desirable features: Because it is the solution to a semidefinite, convex optimization program, its resolution time is very predictable ahead of time, which might be of interest to on-line reconfiguration, or as a first guess for existing methods.[13]

## IV. Conclusions

In this Note, we have considered the control surface allocation problem in the case when the surface allocation is limited to be a linear mapping from moment space to control space. We have shown that an approach to that problem based on ellipsoid volume maximization can be easily recast as a convex optimization problem. This method has been applied to a numerical model of the F-18 HARV and has been compared with other approaches. The convex nature of the optimization problem under consideration makes it possible to incorporate the proposed procedure in a real-time aircraft control allocation reconfiguration in the event of damaged control surfaces. The by-products of the optimization procedure (especially the resulting ellipsoids) may be used in other proposed surface allocation procedures as well.

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# New Constraint Stabilization Technique for Dynamic Systems with Nonholonomic Constraints


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## Introduction

IN the last two decades, computer simulation of multibody dynamic (MBD) systems has enjoyed substantial progress to design and analysis engineering problems such as robot arm manipulators and space vehicles. In general, the dynamic equations of MBD systems with holonomic and/or nonholonomic constraints can be derived and expressed in a set of differential algebraic equations (DAEs). Because the solution procedure of DAEs suffer drawbacks such as constraint violation and numerically stiff in the computer implementation, these have motivated researchers to look for alternative solution procedures that overcome the preceding difficulties. Conventional nonholonomic constraint stabilization techniques such as the stabilization technique by Baumgarte,[1,2] the penalty method by Orlandea et al.[3] and Lötstedt[4] and the penalty staggered stabilized procedure by Park and Chiou[5] are developed to correct the constraint forces during the process of numerical integration. However, these techniques require one to choose suitable stabilization parameters for their different applications. In general, different parameters will cause different results in correcting constraint violations. Moreover, the simulated DAE will become numerically unstable if inappropriate parameters are chosen. To obtain a parameter-free


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